

# Application of Numerical Simulation of Nonlinear models to Three Stages Micro Satellite Launch Vehicles (MSLVs) Trajectory

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**Abstract**— Usually, the analytical solutions to launch vehicles translational motion are implemented through linear models that typically involve solving a set of simultaneous differential equations by numerical methods. In principle, the numerical solutions to trajectory optimization of Launch vehicles are based on point mass model of translational motion where the study of nonlinear effects by such means is by and large avoided. Although there have been many analytical studies of one or another nonlinear effects, the trajectory context is usually idealized or much simplified compared to actual launch vehicle trajectory scenarios. In addition, it is typical for such analytical formulations to be of such complexity as to require numerical evaluation, a situation which negates the values of analysis and actual behaviors. In this study, a novel Simulink based numerical simulation approach for determining variable parameters and non-linear effects of the models of MSLV trajectory waypoints was use to provide calculated time series trajectory variables of MSLV.This approach is assumed necessary for emerging MSLV flight control sensors, stages interface and payload due to their extremely small dimension and lower inertia mass properties. Perhaps the justification for the simulation approach to the solutions of launch vehicles translational motion is the presence of first principle nonlinear equations of motions, discontinuities interpolations and higher order models of the physics of the flight environment.

**Index Terms**—Trajectory, micro satellite, Launch vehicle, flight control, sensor, nonlinear, environment, parameters, waypoint.

## 1 INTRODUCTION

The critical requirement of any MSLV is to reliably deploy microsatellite by propulsive means through a predetermined trajectory from launch point to mission orbit. During the flight, the vehicle motion is guided by program turn so as to steer towards the desired trajectory. The critical trajectory parameters are dependent on the performance of the propulsion system and the approach of turning the vehicle towards desired trajectory. The propulsion system induced acceleration on the vehicle during its motion while the flight program guarantees that optimal trajectory is followed in order to deploy the satellite in the mission orbit. In practice, the dependency of flight path angle on the pitch angle and yaw angle on the body of the vehicle is manipulated through steering actuators in order to implement a program flight. There are many theoretical approaches to determining the optimal trajectories of launch vehicles and can be broadly classified into direct and indirect methods. Direct methods find the optimal control directly and employ only the dynamical and constraint equations. Nonlinear programming [27] and evolutionary methods [22] have been used to solve trajectory optimization problems by the direct method.

Indirect methods solve for the costates of the systems that is the Lagrange multipliers for the system and from the costates derive the controls. Indirect methods require both the dynamical and costates equations to be solved simultaneously. Many methods for solving the indirect method have been studied including gradient methods [31], simulated annealing [33] and genetic algorithms [24]. Genetic Algorithms, Particle Swarm Optimization and Differential Evolution are three of the most known global optimization techniques, but are by no means the only ones. Other global optimization methods include simulated annealing and colony optimization but these were rarely considered for trajectory optimization in the reviewed studies. GA has been used for launch trajectory optimization [25] (sometimes in conjunction with the gradient method [17]) and for multidisciplinary optimization of both trajectory as well as vehicle design [15]. Particle Swarm Optimization is also frequently used for ascent trajectory optimization [22],[19] as is the case for Differential Evolution [22]. Genetic algorithms were most often discussed in the literature but this might be the case because it is the best known method of the three and the most available. In addition, the work of Tuser and Filipic [23] clearly shows that Genetic Algorithm underperformed Differential Evolution by a significant margin when trying to perform a multi objective optimization. It is also stated that it also underperformed Differential Evolution for single objective optimization.

Yunus, 2012 proposed approach of Multi-Criteria Multi-Objective Simulated Annealing (MC-MOSA) algorithm for the design of a launch vehicle for Nano satellites. The algorithm aims to find the optimum trajectory with the optimum design parameters related to aerodynamics and propulsion system as a multidisciplinary optimization to future space transportation vehicle as an alternative to the classic approach to LV design

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proposed by Tsuchiya, T et al, 2002[24] in his study.

In the paper titled "Active Rocket Trajectory Arcs: A Review" published in the journal of Automation and Remote Control by D.M. Azimov in 2005, the author reviews the development of analytical, approximate analytical, and numerical methods for solving the vibrational problem on the determination of optimal rocket trajectories in gravitational fields, and their application to study flight dynamics. Specifics of these methods as applied to solve modern and complex problems are described. A variational problem is formulated and extremal thrust arcs are described. Papers containing results of analytical investigations on thrust arcs are reviewed in depth. Partially investigated problems are described. Problems of great interest in the development of methods for solving the variational problem and problems in the theory of optimal trajectories are mentioned.

John W. H. in 1999 researched on "Low-Thrust Trajectory Optimization using Stochastic Optimization Methods". In his work, he outlined a method for the optimization of low-thrust, interplanetary, spacecraft trajectories. In particular, he described trajectory optimization through the use of stochastic optimization algorithms. The two most widely recognized stochastic methods simulated annealing and genetic algorithms, were utilized. The algorithm developed is useful in producing novel trajectories. The new solutions discovered possessed both non-intuitive structures and very high performance.

Anderson, m et al, 2001 [26] in addition to optimization, considered Aerodynamics and trajectory performance disciplines in his study. Reference vehicle geometry is chosen after all discipline analyses were carried out and a series of parametric trade studies were performed to determine the major vehicle parameters after finalizing the reference vehicle (Stanley, D.O et al, 1992).

In the work of Braun, 1997[23], Trajectory problem is decomposed into sub-problems along domain-specific boundaries (1) Through subspace optimization, each group is given control over its own set of local design variables and is charged with satisfying its own domain-specific constraints (2) The objective of each sub-problem is to reach agreement with the other groups on values of the interdisciplinary variables.

In all previous trajectory solutions, the ordinary point mass differential equations[34] used for optimal trajectory of maybe valid for linear rigid body dynamics model of conventional satellite launch vehicles(CSLVs)but may not be valid for practical implementation since the neglected properties of distributed parameters and nonlinear effects of real flight may become an important factor in dynamic behavior of the MSLV,for these reasons, the existing point mass differential equations will requires an improved solutions with consideration to distributed parameters and centre of mass shift for its suitability to trajectory of MSLV or tailored trajectories design. In addition to these drawbacks, the previous trajectory solutions are not suitable for a variable geometry body.

In this study, due to significant structural flexibility anticipated from the slender body of MSLV, a coupled approach waypoint planning based on distributed instantaneous vehicle mass fraction and multi stage vehicle was integrated into solu-

tion of point mass ordinary differential trajectory equations. The distinct consideration of stages boundary conditions in the trajectory solutions yields a continuously differentiable trajectory definition such that flight path tracking errors and unmodeled disturbances are minimized during flight.

We subsequently developed a novel numerical simulation solutions to higher level models the translational motion that account for behavior of the vehicle properties and fundamental physics of its flight environment using descriptors Simulink block models, analytical models, and nonlinear differential equations in Matlab-Simulink environment.

## 2 MATHEMATICAL MODEL OF THE TRAJECTORY

### 2.1 Motion Geometry

The equations of motion that govern the trajectory Fig.1 of MSLV on the basis of structural mass fraction and centre of mass shift can be conveniently written in terms of its radial distance from the Centre of the earth  $\vec{r}$  and velocity  $\vec{V}$ . In addition, the position vector  $\vec{r}$  is defined by its magnitude  $r$ , its longitude  $\theta_L$  measured from the  $x$ -axis in the equatorial plane, positively eastward and its latitude  $\phi_L$  measured from the equatorial plane, along a meridian and positively northward,  $\gamma$  is the flight path angle and  $\psi_L$  is the azimuth or heading angle measured between positively in the right handed direction about the  $x$ -axis which define the velocity vector during the vehicle motions. These variables form the state vector  $\vec{X} = [r, \theta_L, \phi_L, V, \gamma, \chi]^T$  of the launch vehicle (in spherical and rotating earth coordinates).

The Launch vehicles are propelled and controlled by thrust  $T$  and its corresponding deflection angle  $\alpha$ , small deflections of the thrust vector control (TVC) engines  $\delta$  with respect to their nominal trim positions in pitch  $\delta_p$  and yaw  $\delta_y$  axis and for the first stage of the flight, controlled by deflections of the control surfaces  $\pm\delta_{cs}$ . Therefore inputs to the dynamic model are: Thrust  $T$ , control surface and engine deflections. These variables form the input force and moment vector  $u = [T, \alpha, \delta]^T$  of the launch vehicle.

The effects of perturbations on the vehicle trajectories due to unmodeled vehicle dynamics, Earthary atmospheric rotation and composition, atmospheric forces and moments and wind gusts velocity are also considered. The aerodynamic forces act through the Centre of pressure, the force of gravity acts through the Centre of gravity, and the thrust force is applied through the "Centre of combustion. The Euler angles roll, pitch and yaw ( $\phi, \theta, \psi$ ) define the vehicle attitude [10] with respect to the inertial reference axes. In a launch vehicle the attitude reference is usually measured with respect to the launch pad with the Euler angles initially at  $(0^0, 90^0, 0^0)$  respectively. The vehicle model outputs detectable by sensors are: attitude, attitude rates, rotation angle, angle of attack  $\alpha$  and sideslip angle  $\beta$ .

The assumption of a spherical rotating earth, atmosphere variation of density, pressure and gravity is eminently considered in the study.

The reference axes are shown in Fig.1. with the  $x$  axis is aligned along the fuselage reference line and its direction is positive along the velocity vector. The  $z$  axis is defined posi-

tive downward towards the floor, and the y axis is defined by the right hand rule, perpendicular to the x and z axes and positive towards the right. The equations derived in this studies will consist of three rotational (roll, pitch and yaw), and three translational equations along x, y and z axes. The vehicle forces and moments generated in this model are calculated with respect to the body axes system.

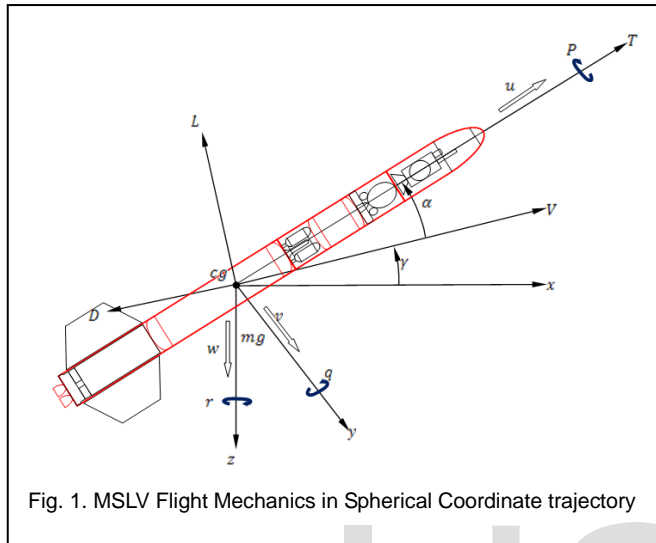


Fig. 1. MSLV Flight Mechanics in Spherical Coordinate trajectory

### 2.2 The Equations of Translational Motion of the MSLV

In this study, we model suitable representative equations of motion and corresponding optimum trajectory suitable for determination of a desired equilibrium of MSLV. Subsequently, the equation is linearized, and stability, controllability, and observability are analyzed. Through nonlinear simulation, we illustrate the extent to which linearized equations approximate the nonlinear ones. Creating flexible, software-defined test platform to validate deployed real-time embedded systems for control, monitoring, and operation.

Trajectory waypoint and guidance law derivation in this study used a point-mass Launch vehicle model of the following form:

$$V = \frac{T \cos \alpha}{m(t)} + \frac{T(1 - \cos \alpha)}{m(t)} - g \sin \gamma(\rho, h, \phi_L) - \frac{D}{m(t)} - \omega^2 r \cos \phi_L (\sin \gamma \cos \phi_L - \cos \gamma \sin \phi_L) \quad (1)$$

$$\gamma = \frac{(T \sin \alpha + L)}{m(t)V} + \left( \frac{V}{r} - \frac{g}{V} \right) \cos \gamma + 2\omega V \cos \phi_L + \frac{\omega^2 r}{V} \cos \phi_L (\cos \gamma \cos \phi_L + \sin \gamma \sin \phi_L) + F_{d\gamma} \quad (2)$$

The instantaneous altitude, longitude and latitude of the vehicle position fig.1 are obtained from the translational equations and defined equations 3a-3d.

$$x = \frac{R_e}{R_e + h} V \cos \gamma \quad (3a)$$

$$h = V \sin \gamma \quad (3b)$$

$$\theta_L = \frac{V \cos \gamma}{r \cos \phi_L} \quad (3c)$$

$$\phi_L = \frac{V \cos \gamma}{r} \quad (3d)$$

$$m \approx -\frac{T}{L_{sp} g} \quad (3e)$$

$$m_0 = m_f + m_{pr}(t_0) \quad (3f)$$

$$m = m(t, X_{cg}) = m_f + m_{pr}(t) \quad (3g)$$

$$m(t, X_{cg}) = m_f + m_{pr} \left( 1 - \frac{T}{1_{sp} m_0} t \right) \quad (3h)$$

$$(0 \leq t \leq t_b) \quad (3i)$$

$$\alpha = \alpha(t) \quad (3j)$$

$$\theta = \gamma - \alpha \quad (3k)$$

$$V(t_b) \quad (3l)$$

$$V(t > t_b)$$

The respective initial conditions depend on the launch time  $t_L$  and on the launch site (identified by the geographical longitude,  $\phi_{ls}$ , and by the latitude,  $\theta_{ls}$ ) ref[1].

These equations are then solved separately for each stage.

In the above equations,  $r$  denotes the distance of the centre of gravity of the vehicle to the centre of the Earth,  $v$  is the modulus of its relative velocity,  $\gamma$  is the flight angle (or path inclination, that is, the angle of the velocity vector with respect to an horizontal plane),  $\theta_L$  is the latitude,  $\phi_L$  is the longitude, and  $\chi$  is the azimuth (angle between the projection of the velocity vector onto the local horizontal plane measured with respect to the axis South-North of the Earth).

The aerodynamic forces consist of the drag force  $D$ , whose modulus is  $0.5\rho(h)SC_D V^2$ , which is opposite to the velocity vector, and of the lift force, whose modulus is  $0.5\rho(h)SC_L V^2$  which is perpendicular to the velocity vector.  $S$  is some positive coefficient (reference area) featuring the engine,  $C_D$  and  $C_L$  and are the drag and the lift coefficients; they depend on the angle of attack and on the Mach number of the vehicle. The drag coefficient  $C_D$  depends on the shape of the rocket and the smoothness of its surface. The drag coefficient is one of the major unknown quantities that are usually determined through wind tunnel or flight test and will be simulated using the correlation of Aerodynamic Drag coefficient relations.

### 2.3 Aerodynamic Forces

Aerodynamic forces are the result of the impact of the environment on the surface of the launch vehicle when it moves. They are defined as the sum of the elementary tangential and normal forces acting on the body of the launch vehicle. Depending on whether the moving body is symmetrical relative to the axis, or its axis of symmetry is directed in the motion

along the velocity vector or deviates from it, there appears one axial force (drag), side force and normal force (lift). The symbol  $D$ ,  $S$ ,  $L$  and donate respectively the drag, side force and normal force depending on the aerodynamics in clean configuration such as:

$$D = \frac{1}{2} \rho(h) S C_D V^2 \quad (4a)$$

$$S = \frac{1}{2} \rho(h) S C_Y V^2 \quad (4b)$$

$$L = \frac{1}{2} \rho(h) S C_L V^2 \quad (4c)$$

Where  $C_D$  is the drag coefficients,  $C_Y$  the side force coefficient  $C_L$  the lift coefficient,  $S$  is the reference surface area for the rocket and  $\rho$  the air density [ref 18].

The coefficient  $C_D$  and  $C_L$  are function of angle of attack  $\alpha$ , Mach number  $M$  and Reynolds number  $R_e$ .

$$C_D = C_D(\alpha, M, R_e) \quad (4d)$$

$$C_L = C_L(\alpha, M, R_e) \quad (4e)$$

Aerodynamics coefficient  $C_D$  and  $C_L$  can be represented in terms of angle of attack  $\alpha$ , Mach number  $M$  and Reynolds number  $R_e$ .

The drag coefficient  $C_D$  depends on the shape of the rocket and the smoothness of its surface. The drag coefficient is one of the major unknown quantities that are usually determined through wind tunnel or flight test and will be simulated using the correlation of Aerodynamic Drag coefficient relations[] in this study as:

$$\text{For } M < 1.3 \quad \text{For } M > 1.3 \quad C_{D\text{MAX}} = 0.067 \text{ at } M = 1.3$$

$$C_D = 0.25 * (1 + M^2) C_D = 0.765 / M^2, M = \frac{V}{a(h)}, a = \sqrt{1.4 \frac{P}{\rho}} \quad (4f)$$

And for the air density  $\rho$ , it decreases with altitude and the influence of drag is greatest at the lower altitudes. For analytical reasons it is convenient to use an exponential approximation to the atmosphere. One such approximation below 9144.0m altitude is given by[11]

$$\rho_h = 1.23366 e^{\frac{h}{9144}} \quad h < 9144m$$

$$\rho_h = 0.034 e^{\frac{h}{6705.6}} \quad h > 9144m \quad (4g)$$

$$P_h = 101325 (1 - 2.25577 \times 10^{-5} h)^{5.25588}$$

$$T = T_0 + \lambda_{tg} h$$

## 2.4 Gravity-Weight

Accurate values of the gravitational acceleration as measured relative to the surface of the earth account for the fact that the earth is a rotating oblate spheroid with flattening at the poles is considered. These values may be calculated to a high degree of accuracy from the 1980 International Gravity Formula, which is

$$g_0 = 9.780327(1 + 0.005279) \sin^2 \phi_L - 0.000023 \sin^4 \phi_L \quad (5a)$$

Where  $\phi_L$  is the latitude and  $g_0$  is expressed in meters per second squared. The formula is based on an ellipsoidal model of the earth and also accounts for the effect of the rotation of the earth. The variation of  $g$  with altitude  $g(h)$  is easily determined from the gravitational law as

$$g = g_0 \frac{R_E^2}{(R_E + h)^2} \quad (5b)$$

## 2.5 Thrust due to Main Engine and Small thrusters

The forces and torque generated are given by Launch vehicle is generated on account of combustion of fuel with mass flow rate and discharge of combustion products through the nozzles.

The thrust  $T$  model at a certain altitude  $h$  with nozzle exit pressure  $P_a$  and pressure at a given height  $P_h$  can be expressed as [1]:

$$T(h) = \frac{m_p}{t_b} v_e + (P_a - P_h) \varepsilon_n A_e \quad (6a)$$

Where  $\varepsilon_n$  is the uncertainty in the precision of nozzle exit area  $A_e$  of the propulsion system of the stage. The interplay among the stated variables determines the thrust profile at sea level. In perfect vacuum (100%),  $P_h = 0$  and thrust reaches its maximum and can be expressed as

$$T_{vac-max} = \frac{m_p}{t_b} v_e + P_a \varepsilon_n A_e \quad (6b)$$

Considering the parameters and the accuracy  $\varepsilon_n$  of the nozzle exit area, the expression for  $T_{vac}(h)$  at any height  $h$  in vacuum shall be modeled as:

$$T_{vac}(h) = T_s(s) = T_{vac-max} - P_h(\varepsilon_n) \quad (6c)$$

Where  $T_s(h)$  is the axial thrust from each stage of the MSLV and emphasize that the thrust is a function of time and height. Specific models of the gravity, the air density, air pressure and the aerodynamic coefficients are implemented for trajectory analysis solution in this study.

## 3 Optimum trajectory Flight formulation for MSLV

The determination of the optimal trajectories leading to insertion of satellites into desired mission orbit is an essential premise to the definition of the guidance strategy, and defines the best performance attainable by an MSLV with specified propulsive characteristics, such as that considered in this study. The considered state variables are velocity, thrust profile, altitude, and mass, whereas the control variable is the programmed angle of attack. The trajectory analysis computes the state variables by solving the equation of motion presented in Eq. 34, and evaluating the constraint conditions at every phase of flight

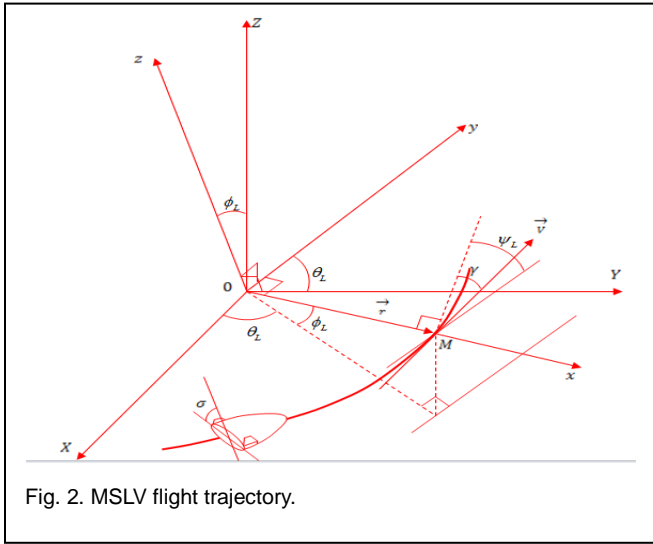


Fig. 2. MSLV flight trajectory.

Significant variable cross section of MSLV imposed a new approach to trajectory design and optimization for its real applications. In this study, a first principle approach of modeling the trajectory flight of MSLV with stochastic parameters is presented in (1),(2). This guidance command was generated on the basis of the set of differential equations, which describes its translational motion as a non-uniform body.

#### 4 Model of Flight Path Angle

A novel tool for trajectory data generation scheme for turning flight and vertical phase on the basis of variable vehicle cross sectional areas is developed from quadratic curve of basic equation of trajectory motion He Linshu (2007).

$$\gamma_t(\mu_i) = \frac{\pi - \gamma_{0i}}{2} \frac{\mu_i^2 - 2\mu_i + 1}{(\mu_{1i} - \mu_{2i})^2} + \frac{\pi - \gamma_{0i}}{2} \frac{\mu_i^2 - 2\mu_i + 1}{(\mu_{1i} - \mu_{2i})^2} \mu_{2i} + \frac{\pi - \gamma_{0i}}{2} \frac{\mu_i^2 + \gamma_{0i}}{(\mu_{1i} - \mu_{2i})^2} \mu_{2i}^2 + \gamma_{0i} \quad (7)$$

Where  $\gamma(\mu)$  is the flight path angle as a function of instantaneous vehicle mass fraction  $\mu$ ,  $\mu_{1i}$  is the mass fraction at time of turning the rocket trajectory from initial value and  $\mu_{2i}$  is the mass fraction at the end of turning to desired final flight path angle  $\gamma_0$ .  $i$  is the flight profile of stages.

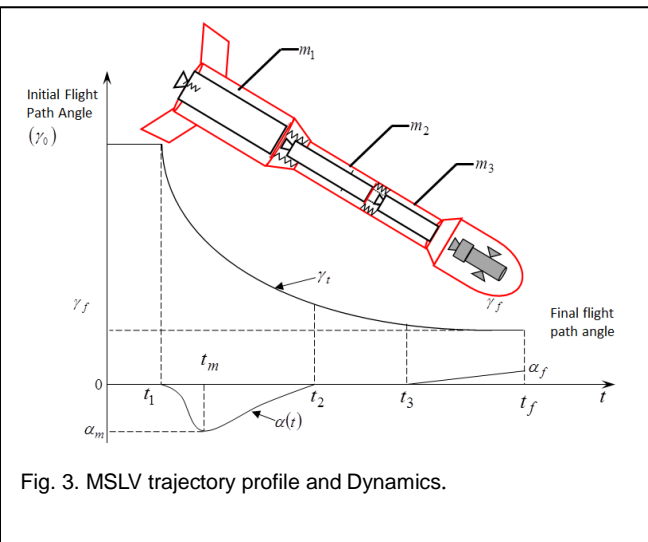


Fig. 3. MSLV trajectory profile and Dynamics.

Adapting the MSLV as research object, and according to improved MSLV vehicle dynamic models, the waypoints of launch vehicle motion was established and generated in Matlab/Simulink on the basis of additional trajectory flight path model equations stated above. The non linear differential equations and the boundary conditions required are specified by the respective Simulink codes and results can be plotted for easy analytical predictions. From this derived model, several plots were made which describes the characteristics of the various parameters as encapsulated in the general equation governing the trajectory variables.

The essential parameters considered as solution to the waypoint planning include the vehicle stages mass, model of the atmosphere, propulsion system, vehicle mass fraction geometric variation, coefficient of drag as a function of march numbers, gravity variation as a function of latitude and altitude, earth rotation effect and stages separations non-linearity.

Adapting the MSLV as research object, and according to improved MSLV vehicle dynamic models, the waypoints of launch vehicle motion and the Launch Vehicle steering command was established and generated in Matlab/Simulink. The differential equations and the boundary conditions required are specified by the respective Simulink codes and results can be plotted for analysis. The nominal trajectory for a given mission is pre-computed and stored on-board. The nominal pitch rate is continuously updated by the guidance system such that it always equals the rate of change of flight path angle. The variation of the flight path angle during insertion flight has substantial influence on the injection accuracy in orbit, acceleration loads, and final orbital velocity. It is influenced by a programmed angle of attack.

#### 5 Initial Conditions

For a typical solid propellant LV orbital payload insertion, the trajectory starts from the launch site with initial altitude at sea level, the initial velocity of the launch site as a contribution to velocity gain and, the initial condition is also that the flight path angle should be  $\gamma_0=90$  degrees, and the initial the angle of attack also should be zero degrees.

In the equation of motion, the steering angle of the thrust,  $\alpha$  is the only control variable to point and direct the rocket into desired direction. The lifting of rocket vertically corresponds to a steering angle of 90 degree and as the rocket achieves higher altitude, it begins to perform pitch over so that  $\alpha$  is less than 90 degree, eventually, the rocket is travelling with  $\alpha$  near zero.

The performance index to this optimization is steering law,  $\alpha(t)$ , which is now modified as a function of time and variable cross sectional area so as to minimize its elastic behaviour

The new function is a key component of algorithms for GNC system because it essentially equivalent to determining the trajectories and controls that transfer MSLV from Launch site to mission orbit.

The mass and propulsive thrust is partition in the numerical solutions as follows:

$$m = \begin{cases} m_{0,1} - \dot{m}_{p,1} \alpha_f & \alpha_f < t_{b,1} \\ m_{0,3} - \dot{m}_{p,3} (\alpha_f - t_{b,1}) & \alpha_f < t_{b,1} + t_{b,2} \\ m_{0,3} - \dot{m}_{p,3} (\alpha_f - t_{b,2} - t_{b,1}) & \alpha_f < t_{b,1} + t_{b,2} + t_{b,3} \end{cases}$$

$$T = \begin{cases} T_1 & \alpha_f < t_{b,1} \\ T_2 & \alpha_f < t_{b,1} + t_{b,2} \\ T_3 & \alpha_f < t_{b,1} + t_{b,2} + t_{b,3} \end{cases} \quad (8)$$

### 6 Application of Numerical Simulation of Nonlinear models to MSLV Trajectory

Usually launch vehicles translational motions are described by linear models that typically involve solving a set of simultaneous, (14) nonlinear partial differential equations by numerical methods. In principle, the numerical solution to trajectory optimization of Launch vehicles is based on point mass model of translational motion where the study of nonlinear events by such means is by and large intractable. Although there have been many analytical studies of one or another nonlinear effects, the system context is usually idealized or much simplified compared to realistic launch vehicle trajectory scenarios. In addition, it is typical for such analytical formulations to be of such complexity as to require numerical evaluation, a situation which negates the values of analysis—insight and generality. Perhaps the justification for the simulation approach to the solutions of launch vehicles waypoint planning in this study is the presence of nonlinear models of equation of motions and properties of the flight environment.

A novel Simulink based numerical simulation approach for determining variable parameters and non-linear effects of the models of MSLV trajectory waypoints is used to provide calculated time series trajectory of MSLV. This approach is assumed necessary for emerging MSLV flight control sensors, stages interface and payload due to their extremely small dimension and lower inertia masses.

#### 6.1 The Nonlinear simulation scheme of MSLV

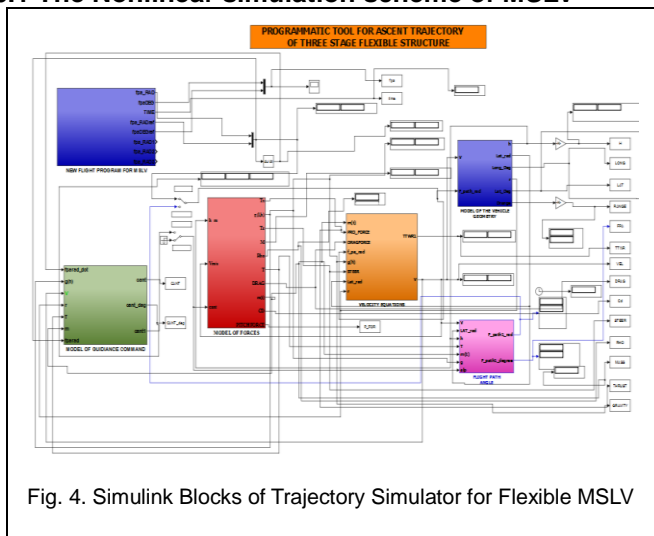


Fig. 4. Simulink Blocks of Trajectory Simulator for Flexible MSLV

#### 6.2 Stage waypoint variables for trajectory profile of MSLV.

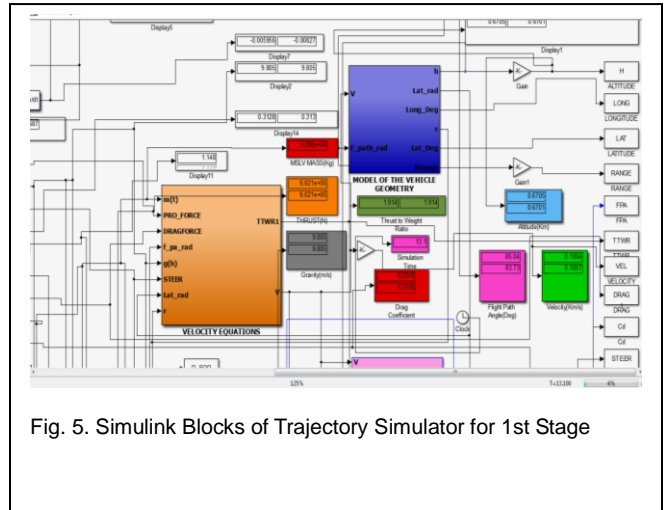


Fig. 5. Simulink Blocks of Trajectory Simulator for 1st Stage

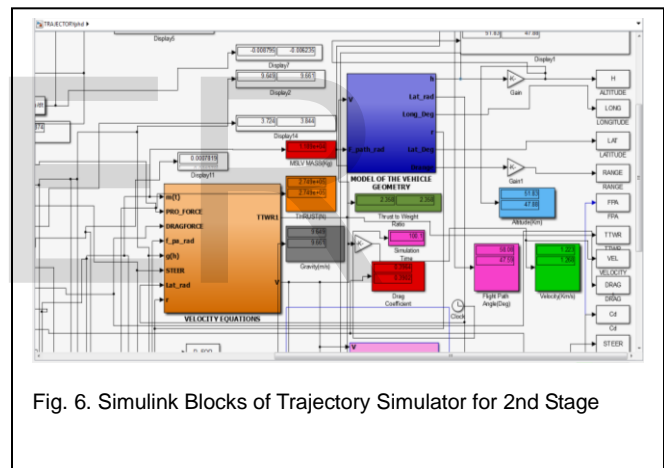


Fig. 6. Simulink Blocks of Trajectory Simulator for 2nd Stage

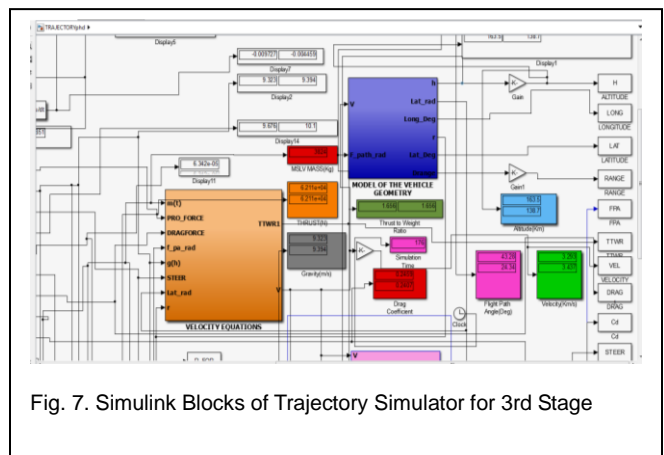
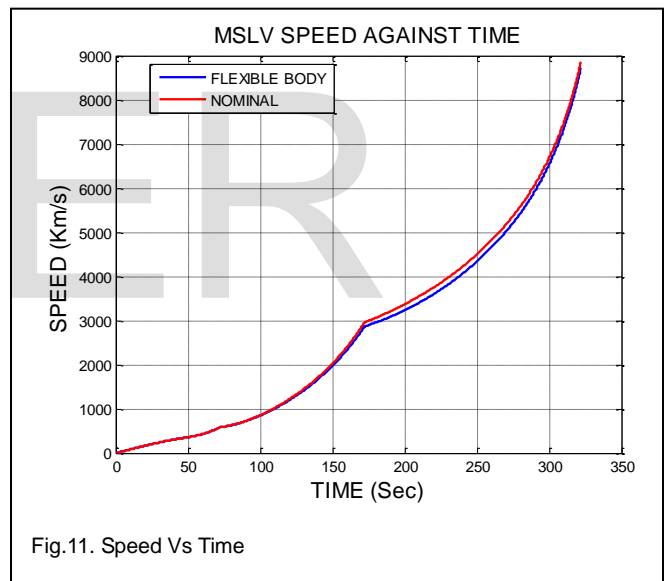
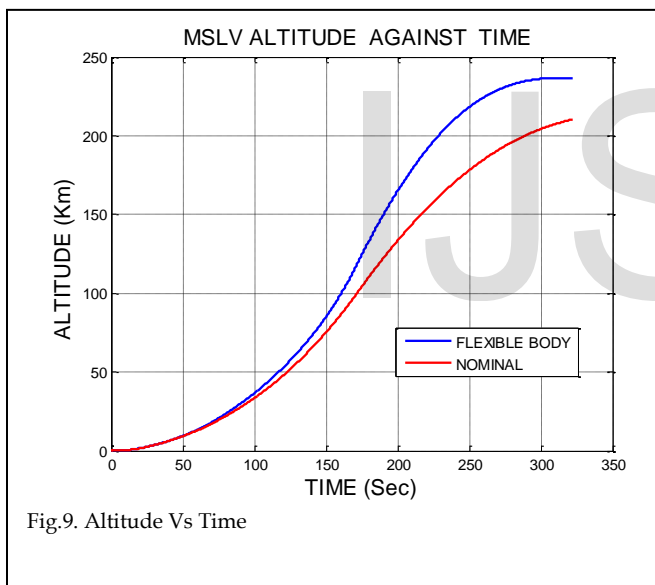
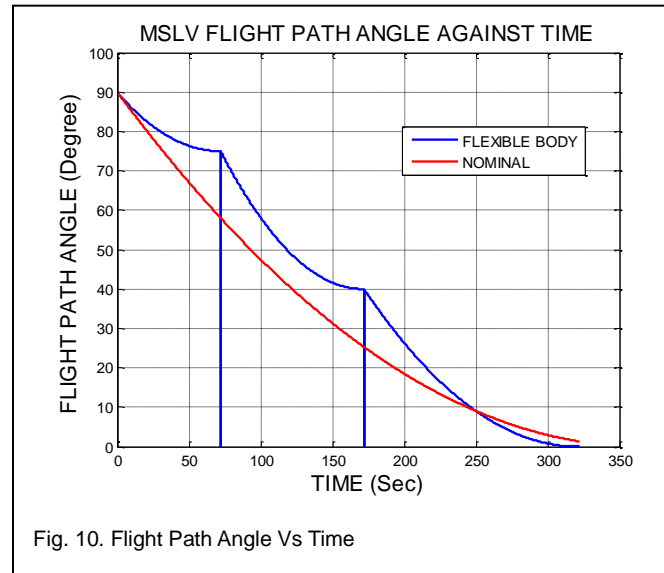
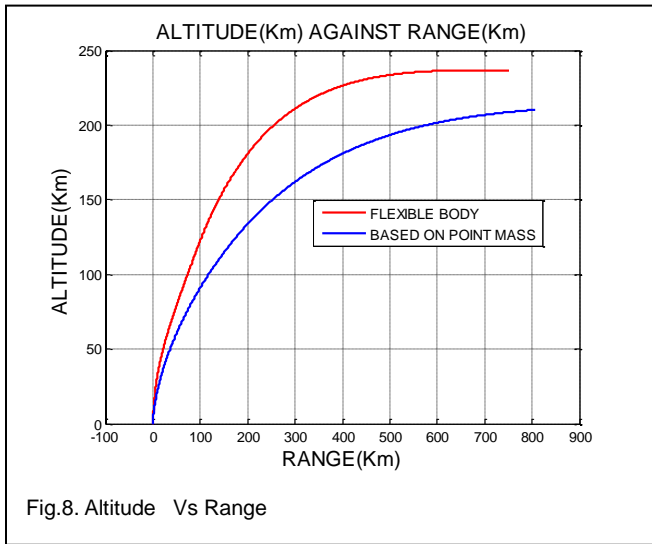


Fig. 7. Simulink Blocks of Trajectory Simulator for 3rd Stage

### 6.3 Simulink Simulation results of the flight trajectory

The results obtained are presented in the curves shown below in Fig. 8-15.



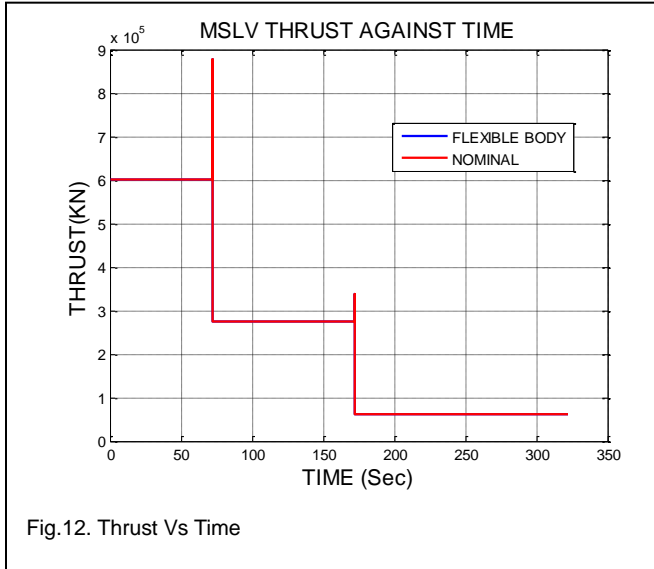


Fig.12. Thrust Vs Time

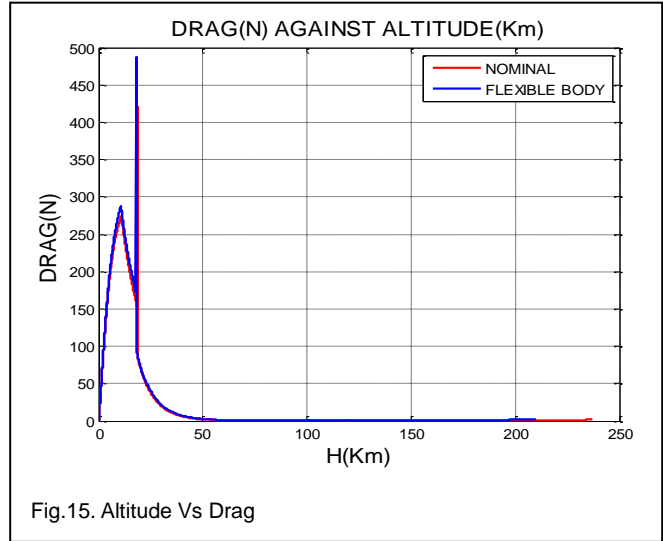


Fig.15. Altitude Vs Drag

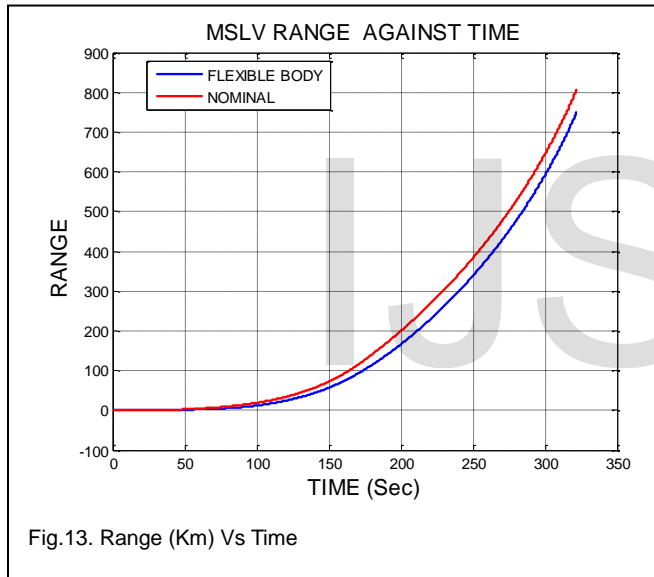


Fig.13. Range (Km) Vs Time

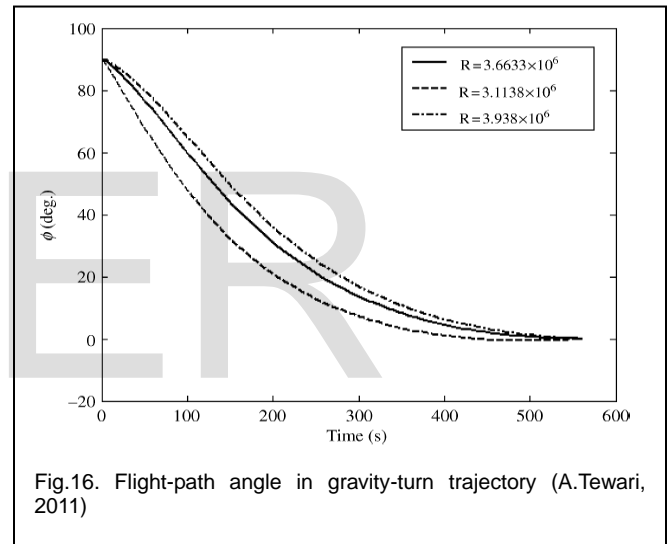


Fig.16. Flight-path angle in gravity-turn trajectory (A.Tewari, 2011)

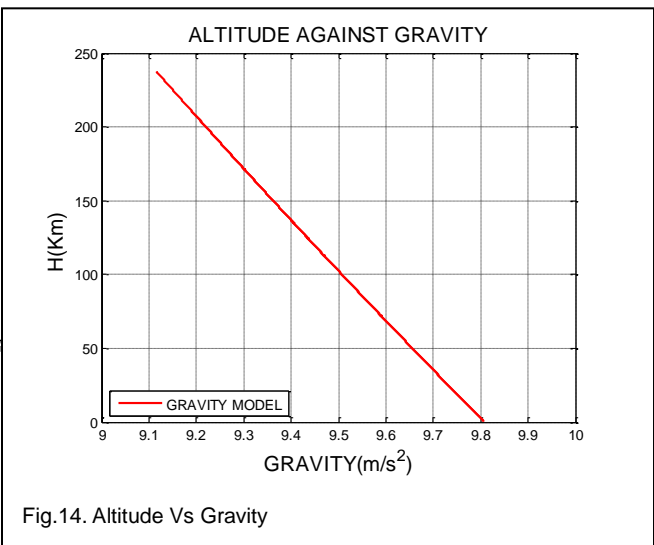


Fig.14. Altitude Vs Gravity

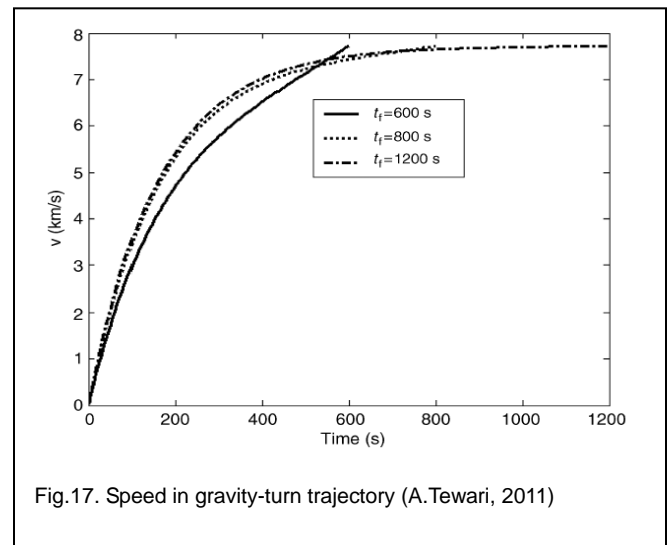


Fig.17. Speed in gravity-turn trajectory (A.Tewari, 2011)



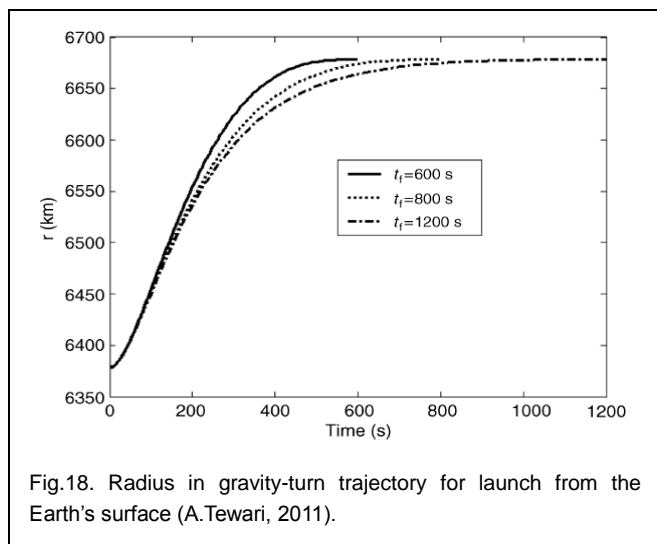


Fig.18. Radius in gravity-turn trajectory for launch from the Earth's surface (A.Tewari, 2011).

#### 6.4 Observation and Discussion of Results (Fig.8-18)

In this study, we have presented simulation solution of nonlinear trajectory motions and environment of MSLV with the single greatest justification for the approach to launch vehicles waypoint planning in the presence of nonlinear elements. We have also demonstrated the realistic waypoint planning of the proposed solution by simulating the behaviour of this vehicle via variable mass fraction solution approach by application of Matlab/Simulink to Runge Kutta numerical solutions and flight environment nonlinearities.

The results compare favourably with existing analytical trajectory optimization techniques but also reveal the anticipated practical behaviours as obtained in actual vehicle flight. This solution can serve as a basis for control engineers to correct the trajectory due to model errors and unmodeled disturbances for integration into guidance and navigation tasks. The result of this solution scheme also reveals extra large transverse displacement and need for bending modes control due to sudden change of flight path angle during stages separation. The plots (Fig.8-15) validated the output of this tool because of its compatibility with realistic behaviour of VEGA Launch Vehicle Ref: Vega User's Manual(2014)Source: www.arianespace.com. The profile of our result also agrees with results in literature as shown in fig. 4.21 page 224 (A.Tewari, 2011) and reveals trajectory errors of fig.4.27.page 226 (A.Tewari, 2011) as shown in fig.16.

#### 7 Conclusion

In this study we developed improved Simulink based path planning algorithm for three stage Launch Vehicle on the basis of nonlinear translational equations, physics of the atmosphere, solution approach of He Lishu, 2007[] and non-rigid body using parameterization results of Adetoro et al,2014[]. In results, we generate a nominal trajectories tools of a flexible flying vehicle for various trajectory problems as a solution to two-point boundary value problems. The result of the nonlinear trajectory simulation tool revealed the concealed discontinuity in flight path, at separation and anticipated damages to

control loops for a significant vehicle elastic displacement. This tool demonstrated the importance of including flexibility and variable point mass structure into the coupled effect of slender body during trajectory motion. For the general case, the guidance system must maintain the vehicle on the nominal trajectories. This study can be applied for path planning for tailored trajectories for microsatellite deployment as well as missile defense type applications.

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